Foreword

This is an extraordinary book. The senior author (Elwyn Berlekamp) plays Go at only the 10-kyu level, and his colleague David Wolfe is rated an amateur shodan, yet they have developed techniques to solve late-stage endgame problems that stump top-ranking professional players. The problems typically offer a bewildering choice of similar-looking moves, each worth only one or two points, but with subtle priority relationships that cannot be adequately described by sente and gote. The solutions come out of combinatorial game theory, a branch of mathematics that Berlekamp helped develop. A Go player who masters its techniques can extract a one-point win from positions where the uninitiated will almost invariably lose or draw.

1.3 Teaser

After reading this book, the industrious reader will be rewarded by being able to solve problems such as those in Appendix C, one of which appears in Figure 1.6. The object is to play all of the small endgame moves in just the right order to get the last point. This problem has stumped several 9-dan professionals from Japan and China, and no one has solved it without the techniques from combinatorial game theory. The diligent reader who masters the relevant mathematics will understand the solutions in Appendix D.

If you are a strong amateur Go player, and believe this problem looks easy, you are not alone. Unfortunately, there is little that can be said briefly to convince you otherwise. One property of this problem that differs from most in the Go literature (e.g., [Nak84]) is that there is no one key move; all of the first twenty or more moves are equally difficult, and on each of these moves very few choices lead to a winning position. So, even if a Go player thinks he has the answer, verifying it requires a human or computer opponent well versed in the theory.

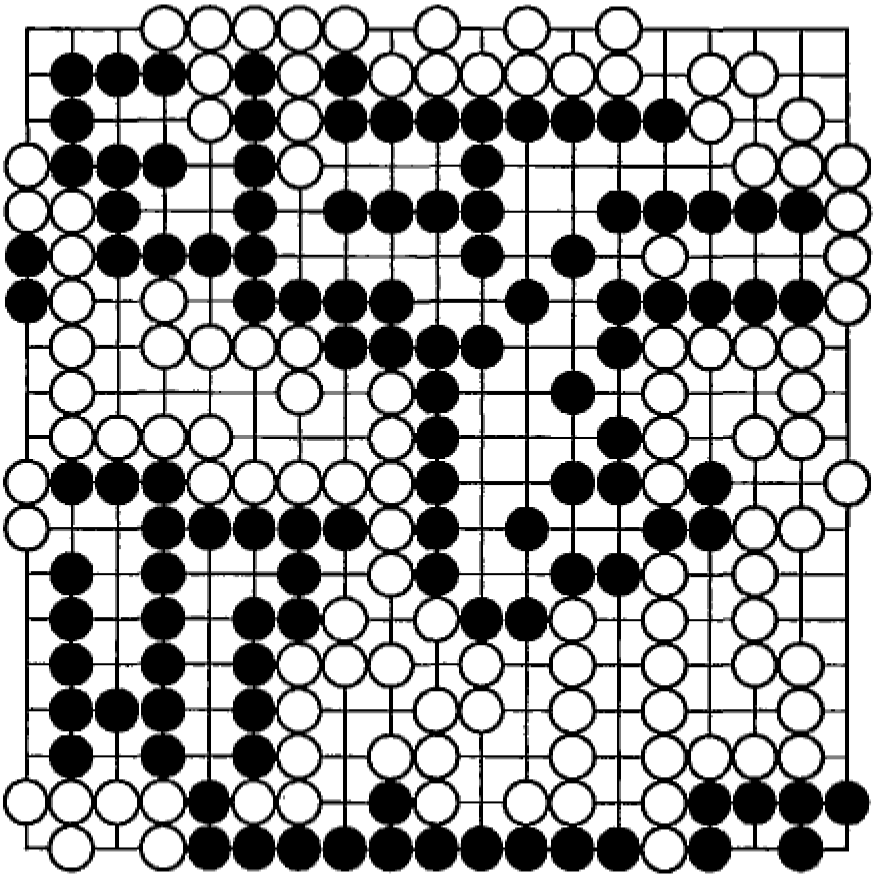


Figure 1.6: *White to move and win*

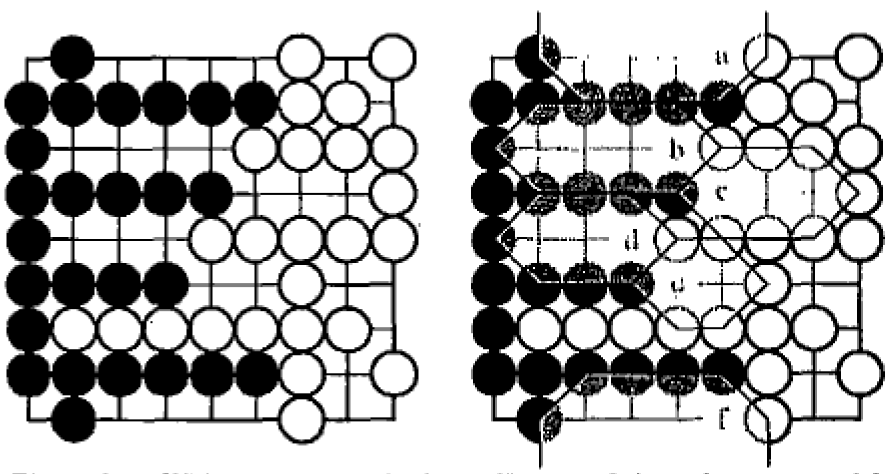


Figure 2.1: *White to move and win — How much is each move worth?*

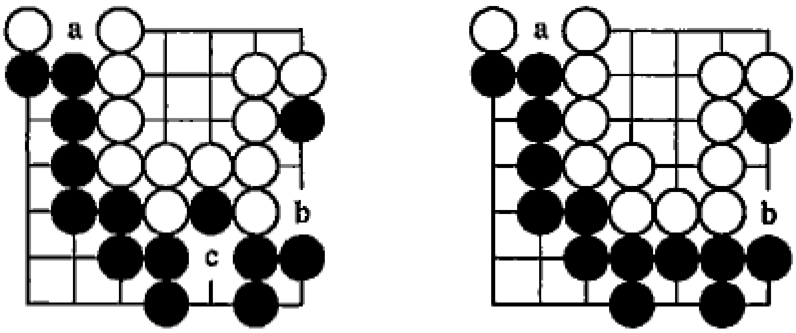
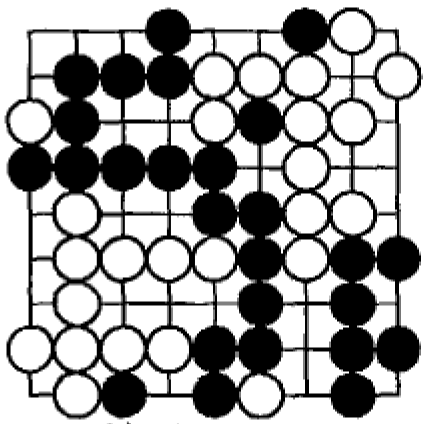
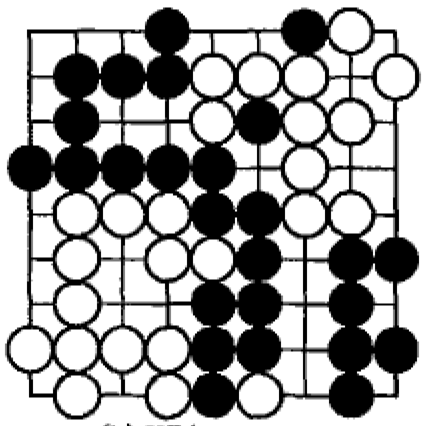


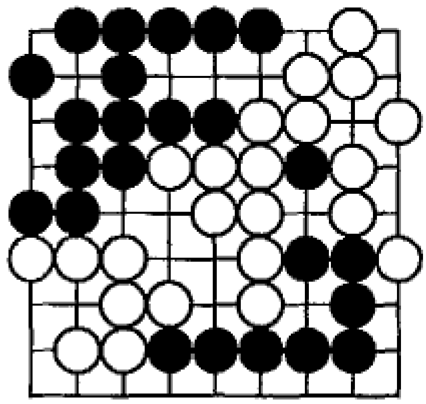
Figure 2.5: *White to move and win. (Both positions are identical except for the region around* c.



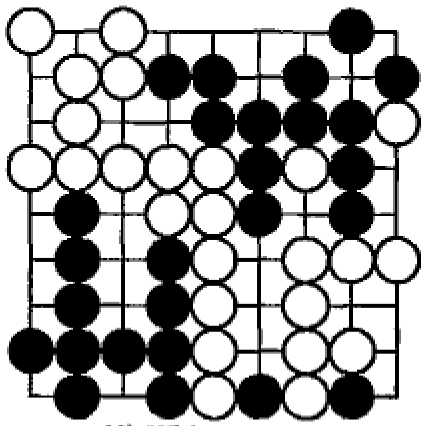
(a) White to move



(b) White to move



(c) White to move



(d) White to move

Figure 3.1: *Four small endgame problems*

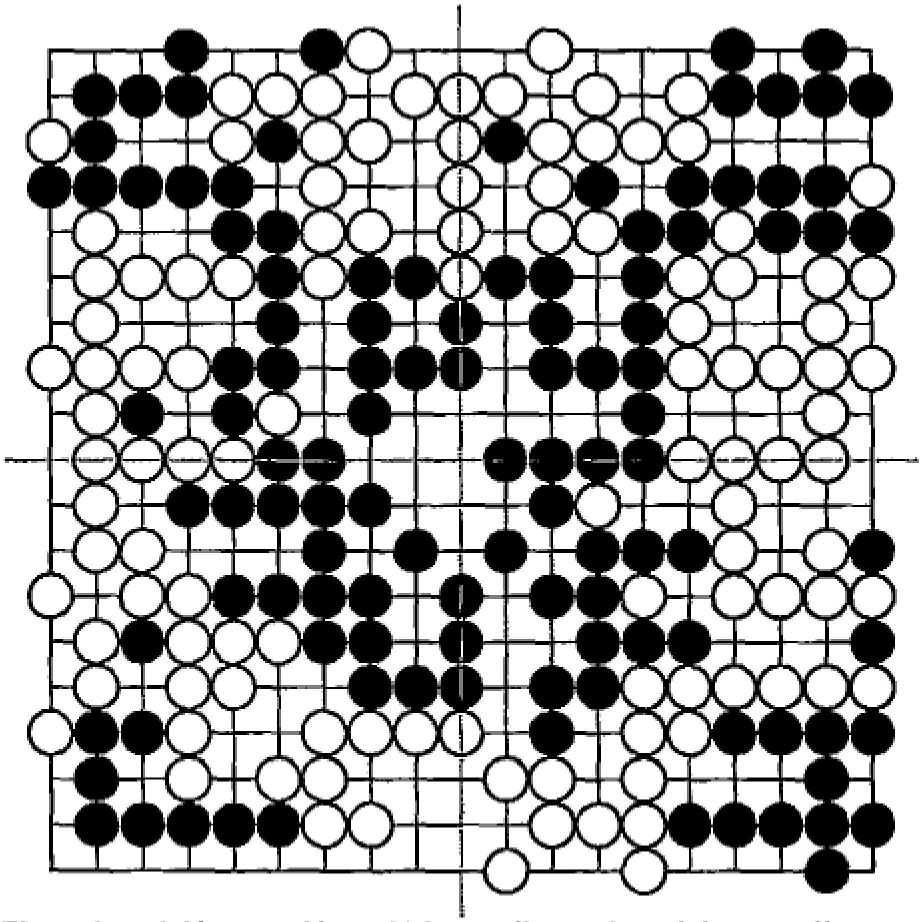
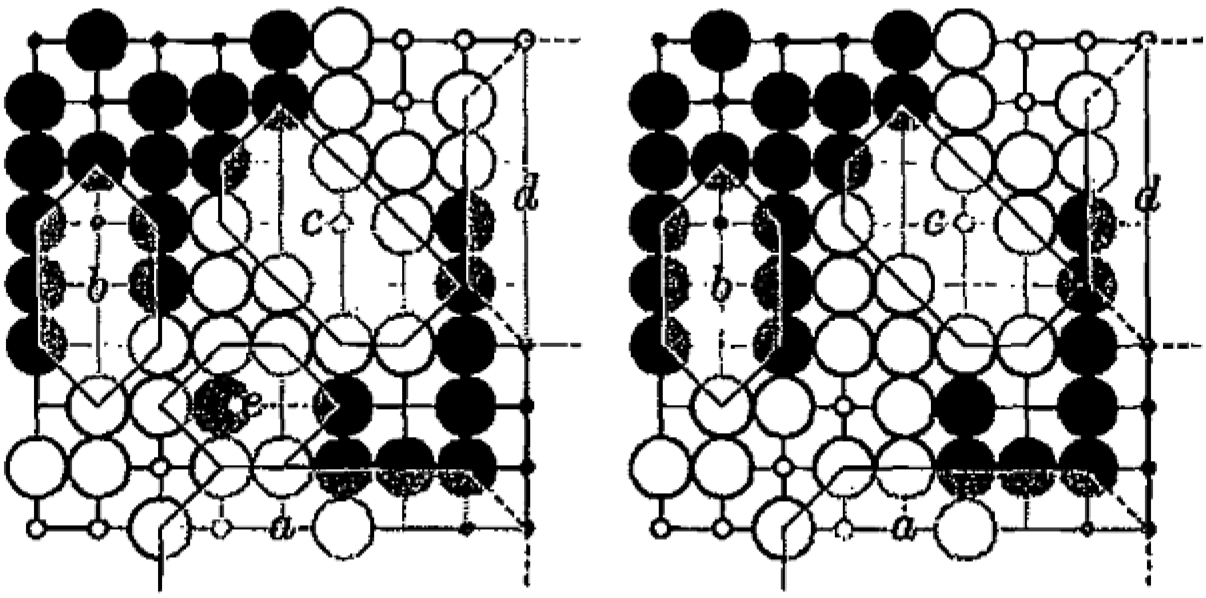
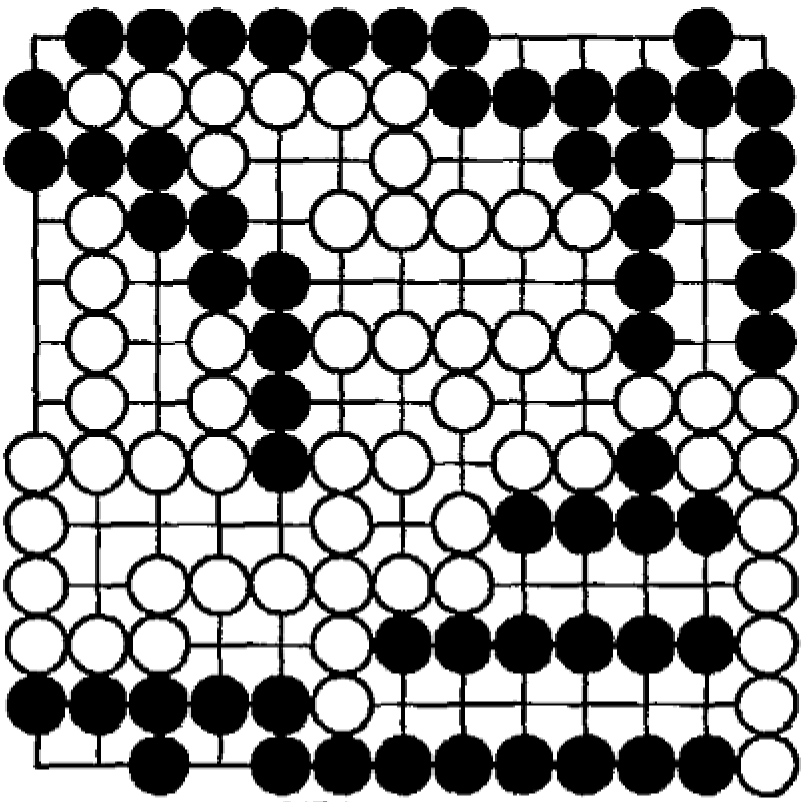


Figure 3.2: *A bigger problem which actually consists of the 4 smaller problems pasted together*



White to move White to move

Figure 4.2: *Two related endgame problems*



White to move

Figure 4.14: *A problem requiring Theorem 8*

4.9 9-dan stumping problem

The next problem we solve is the first problem given in Section 1.3. This problem is sufficiently challenging to stump every professional player who has tried it, including several 9-dan's from Japan and China.[3] Figure 4.17, along with the table in Figure 4.18 summarize the analysis. A more condensed form of the solution is shown in Figure 4.19. (As in the last Section, the Figure fails to maintain the integer part of the game values, but incentives, and therefore optimal play, can be found.)

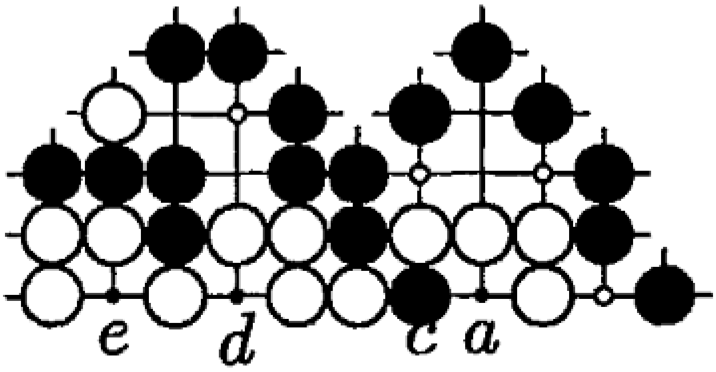
In some cases it is not clear at first that the theorems apply; for instance, one must verify the involvement of the corner near N is of no consequence to the canonical form. So, this problem also demonstrates the robustness of the results.

By Japanese scoring, the score is $0 + \int g$, where g is infinitesimal and has uppitiness zero, but g is negative, and White can win if he plays properly. The incentives chart in Figure 4.5 aids in finding the dominant moves, the first one for White being at C.

This problem is equally challenging if Chinese scoring is used (where occupied plus surrounded territory is scored, rather than surrounded territory plus prisoners). If White wins by one point by Japanese scoring, the number of points on the board after dame are filled will be odd, and so White will fill the last dame. Therefore, White will also win by Chinese scoring. If, however, White ties by Japanese scoring, the score must still end up odd, so since Black made the first move of the game, Black will get the last dame and win. More discussion of why the mathematics usually applies to *both* Chinese and Japanese scoring may be found in Appendices A and B, especially Section B.3.7.

4.10.3 A "realistic" example

Although the position is contrived, the following example demonstrates how multiple sockets might look in a game.



Recall that very short White corridors can have Black markings as in Section 4.3. Here, a, c, d and e are all sockets. (Black's play at a captures White's stone at c, and then Black can play at c to connect her group.) The stone at c is not marked since the effects of being both a captured stone (marked white) and a socket (marked black) cancel. There is one unblocked corridor of value 1, and blocked corridors of values 1, 1, 1, and 1/2. One of the sockets mates with the $u = 1$, and the other three are unmated. The blocked corridors of value 1 associate with the unmated sockets, saturating them, and the last $b = 1/2$ associates with socket of value $u = 1$. The value of the position is $u/2 + bu/2 = 3/4$.

A.1 Rulesets can (rarely) yield differing results

Consider next the game shown in Figure A.2. It is now Black's move. If both players play competently for the rest of the game, then what will be the final difference between Black's score and White's score? Who won? The table of net scores in Figure A.3 shows that your answer depends very much on your particular dialect of rulespeak.

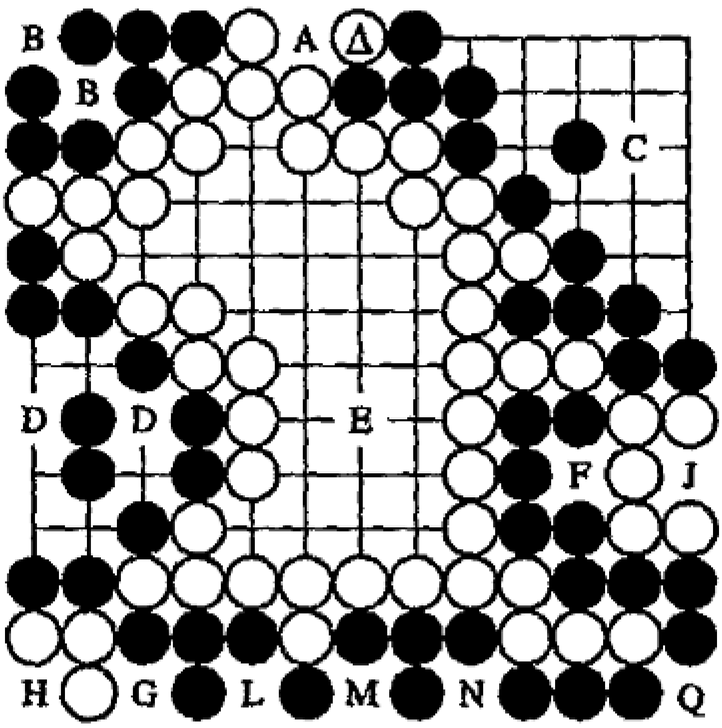


Figure A.2: *What is the score? White has just played at* Δ*, capturing a single black stone in ko. Each side has 1 captive, and 52 stones on the board, none of which are dead.*

Region	Japan	China	Ing 1986	Ancient	Math
A	0	-1	-1	-1	$\surd$
B	$+2$	$+2$	$+2$	0	$\surd$
C	$+18$	$+18$	$+18$	$+16$	$\surd$
D	$+8$	$+8$	$+8$	$+6$	$\surd$
E	-28	-28	-28	-26	$\surd$
F	0	0	$\frac{2}{4}-\frac{2}{4}=0$	0	0
G	0	0	$\frac{2}{3}-\frac{1}{3}=\frac{1}{3}$	0	0
H, J	0	-2	-2	0	0
L, M, N, Q	0	$+4$	$+4$	$+4$	$+4$
TOTAL	0	$+1$	$+1\frac{1}{3}$	-1	

Figure A.3: *Score of Figure A.2 under different rules. A $\surd$ indicates that mathematical rules can be designed to match any of the scoring methods.*

Appendix C

Problems

This Appendix contains problems which can be used for practice. Appendix D provides the values for each region from which you can deduce dominant winning lines using the information summarized in Figure E.9. Possible winning first moves are also indicated in the solution diagrams. Keep in mind, however, that simply knowing the first move is only a very small part of truly understanding the solutions to most of the problems. You can try out your solutions against Chen's software [Che] discussed in Section 1.4.

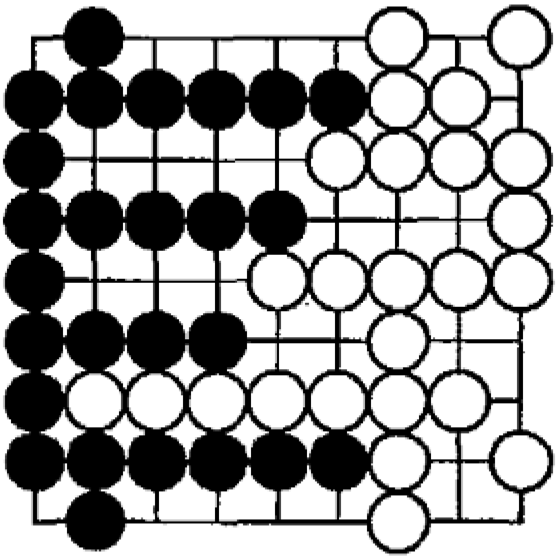


Figure C.1: *Numbers warmup*

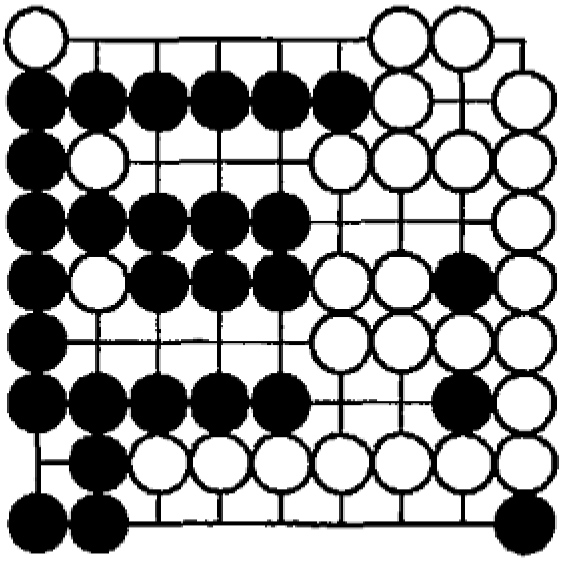


Figure C.2: *Ups and stars warmup*

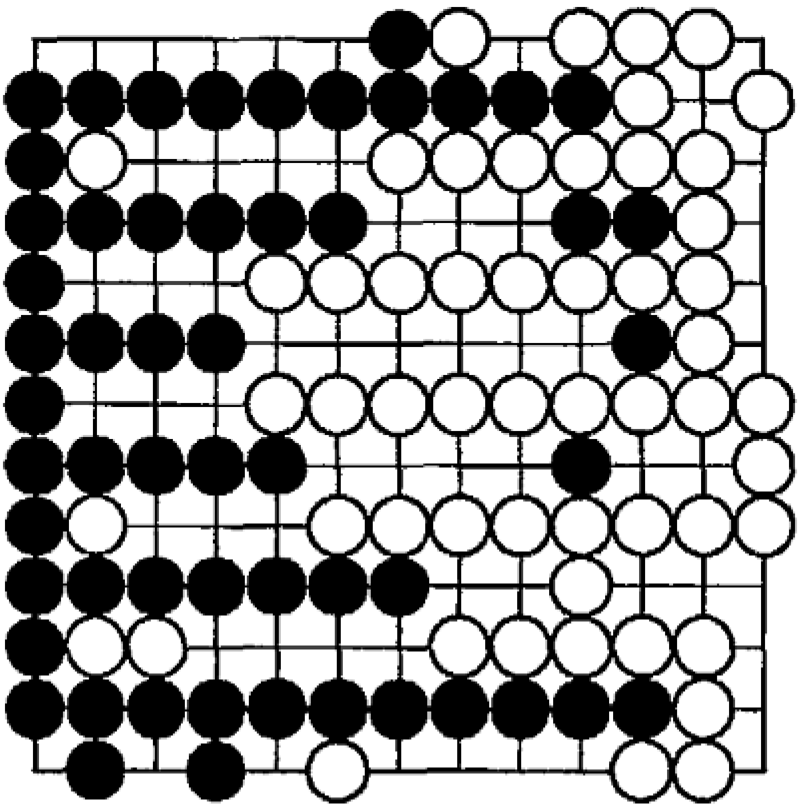


Figure C.3: *Combination of* $0^n|x$ *warmup*

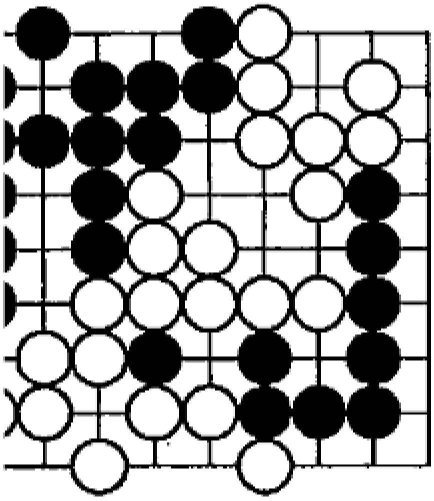


Figure C.4: *#'s, ↑'s and ∗'s*

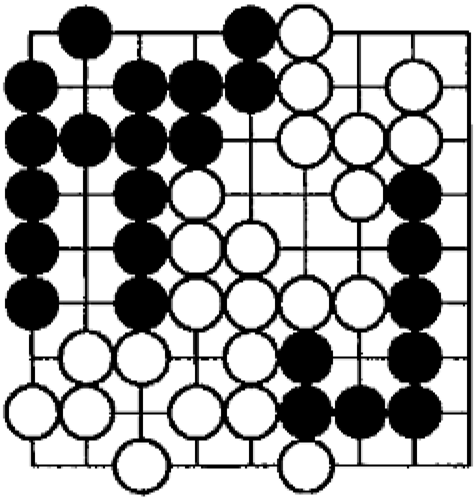


Figure C.5: *#'s, ↑'s and ∗'s*

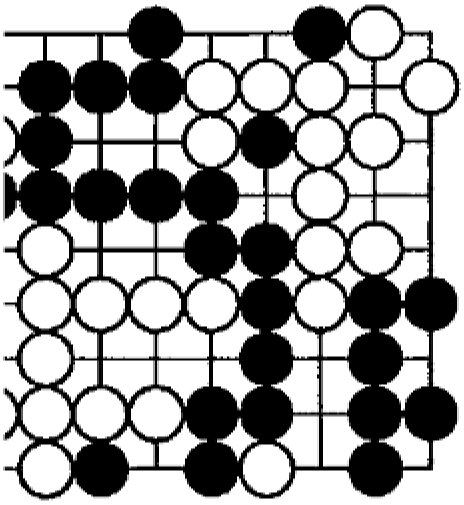


Figure C.6: *#'s, ↑'s and ∗'s*

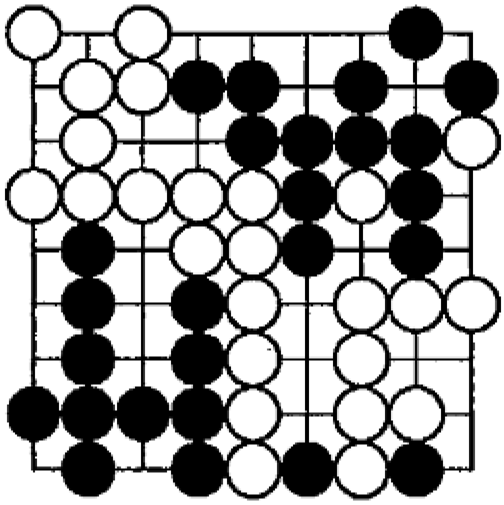


Figure C.7: *#'s, ↑'s and ∗'s*

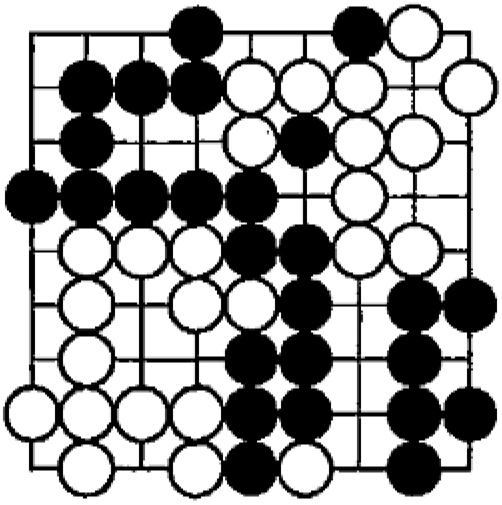


Figure C.8: *#'s, ↑'s and ∗'s*

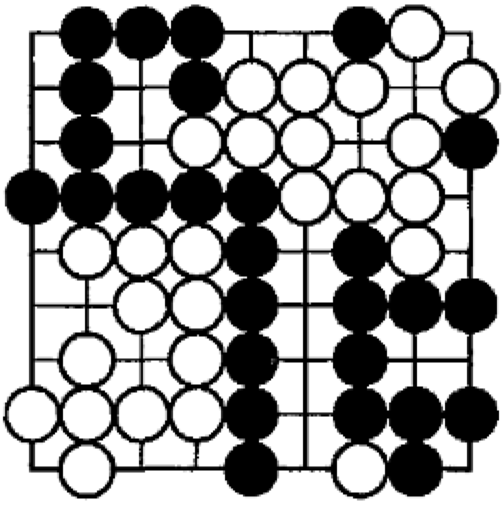


Figure C.9: *#'s, ↑'s and ∗'s*

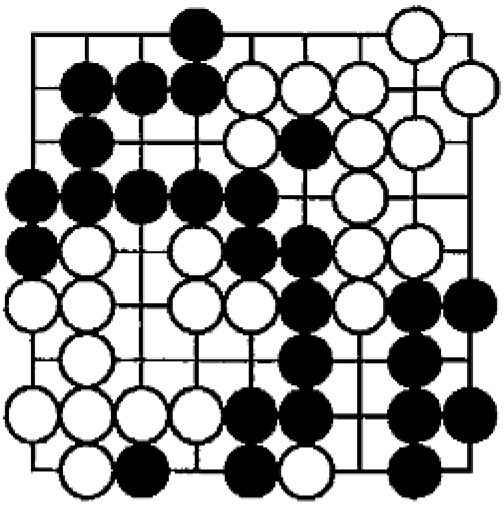


Figure C.10: *#'s, ↑'s and ∗'s*

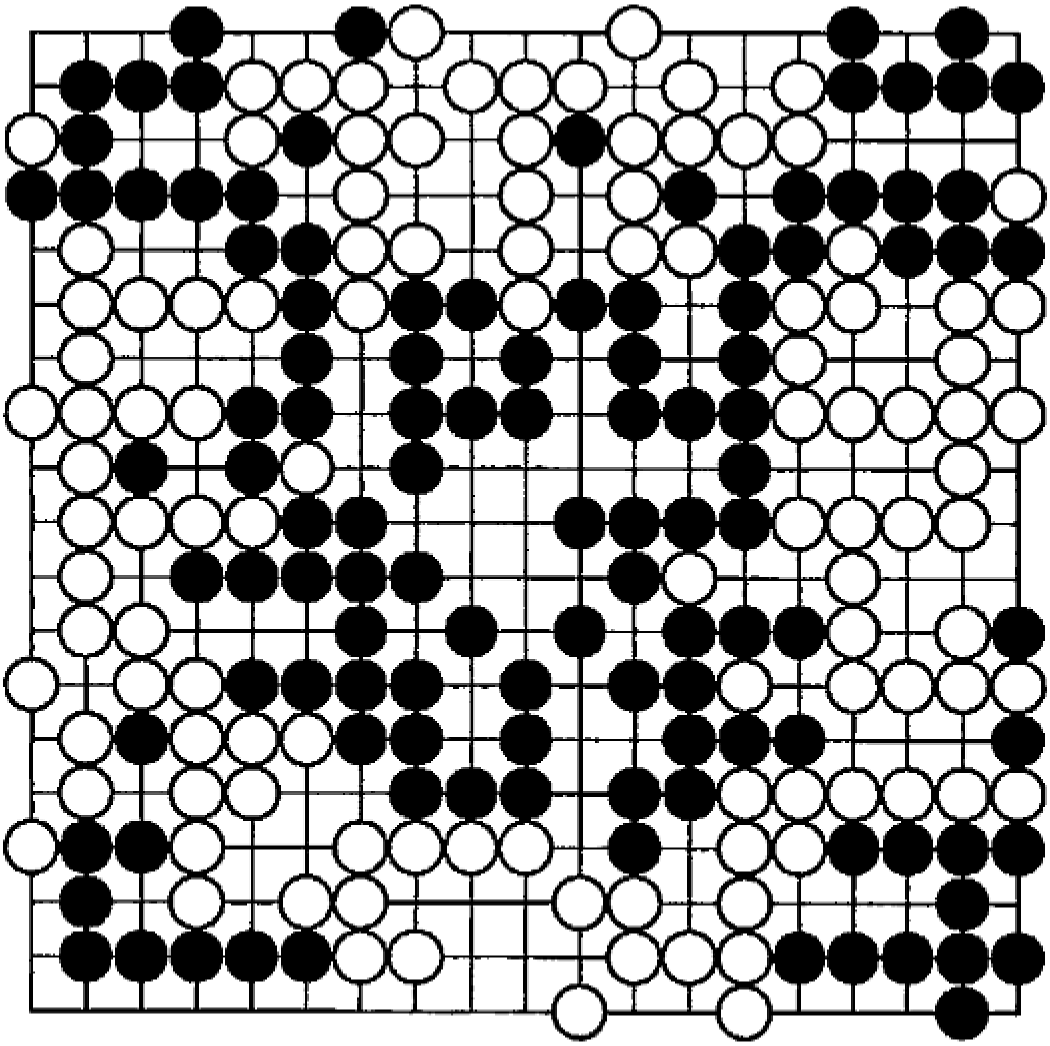


Figure C.11: *#'s, ↑'s and *'s*

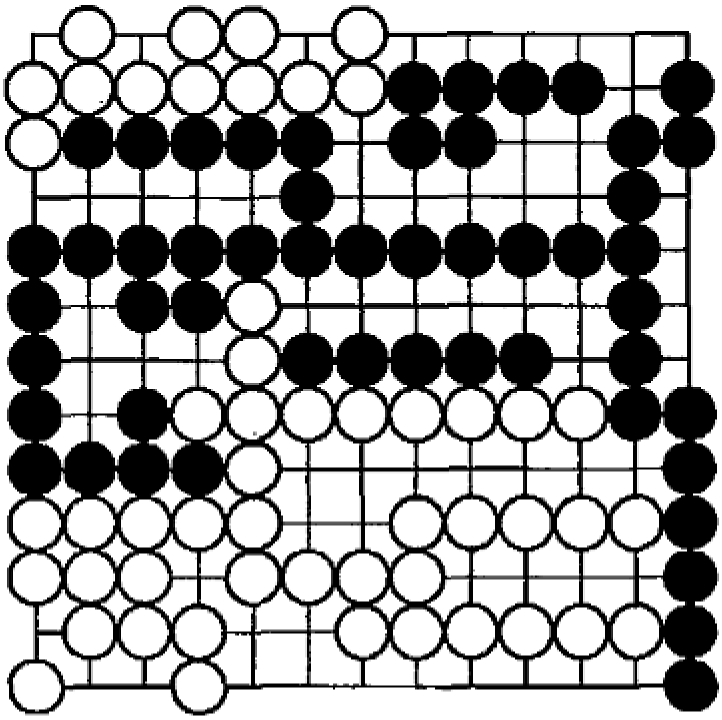


Figure C.12: $0^n|x$'s

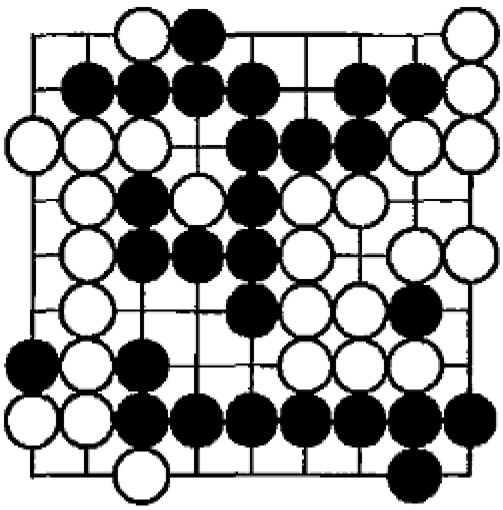


Figure C.13: $0^n|x$'s

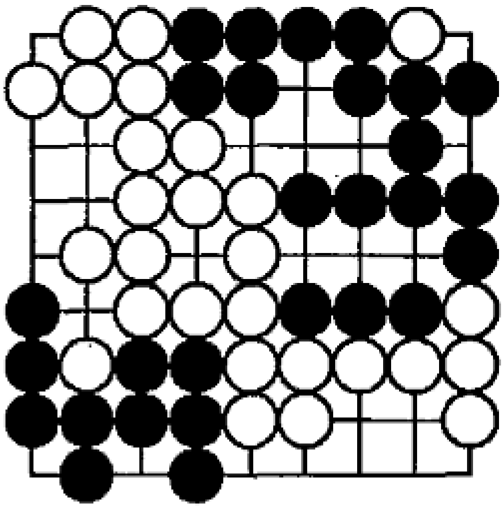


Figure C.14: $0^n|x$'s

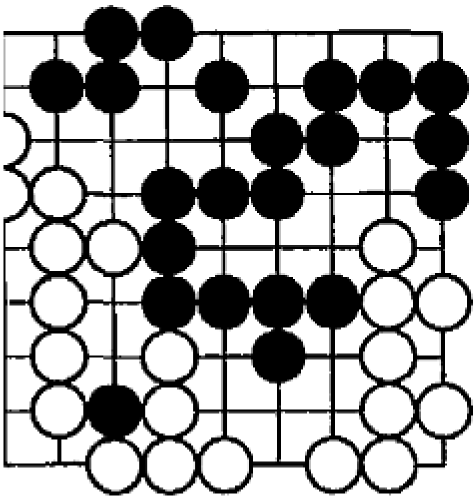


Figure C.15: *More* $\uparrow$ *'s and* $*$ *'s*

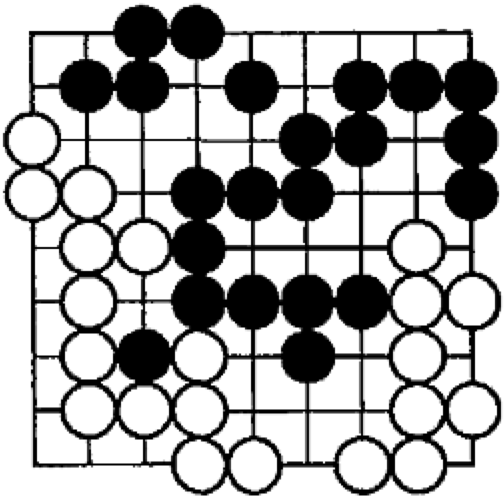


Figure C.16: *More* $\uparrow$ *'s and* $*$ *'s*

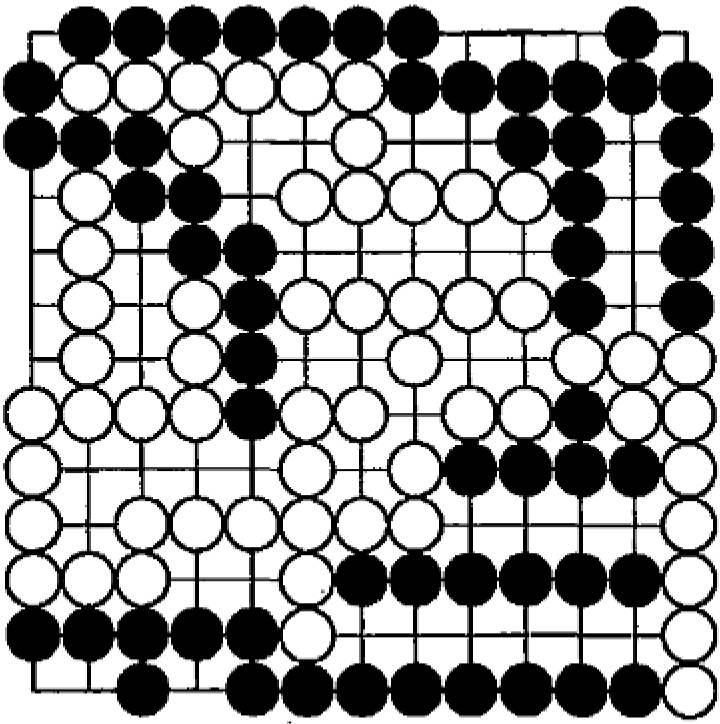


Figure C.17: *Multiple corridors*

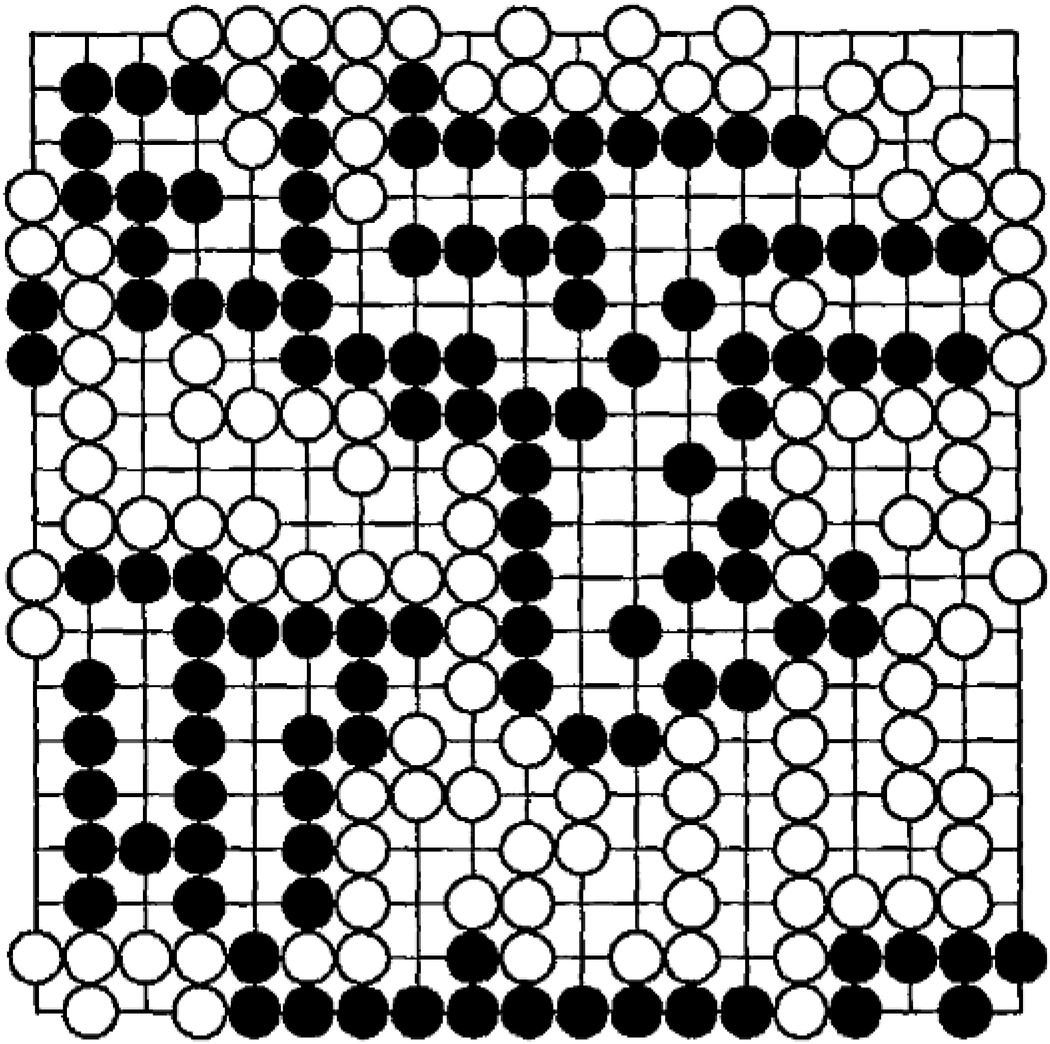


Figure C.18: *Multiple corridors*

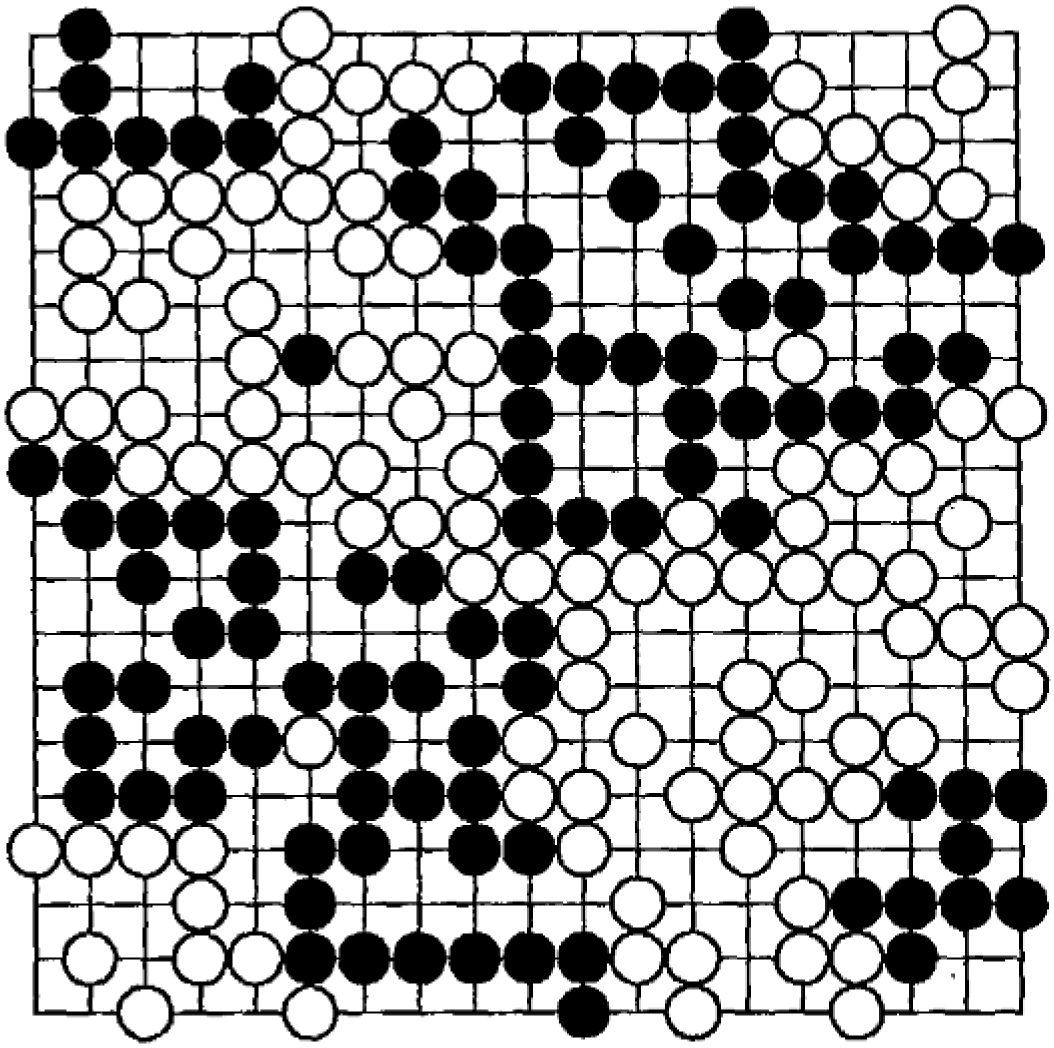


Figure C.19: *Lots of* $0^n|x$*'s*

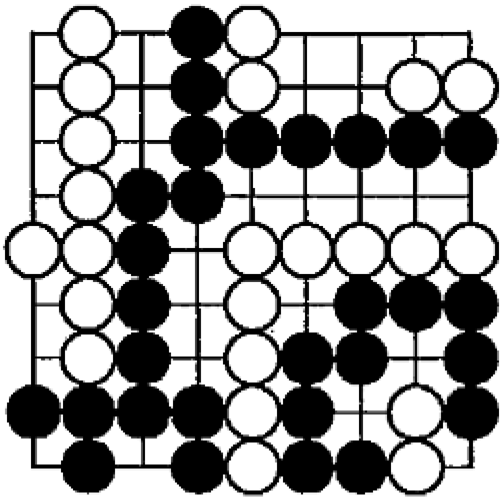


Figure C.20: *Capturing race? Five white captives are off the board.*

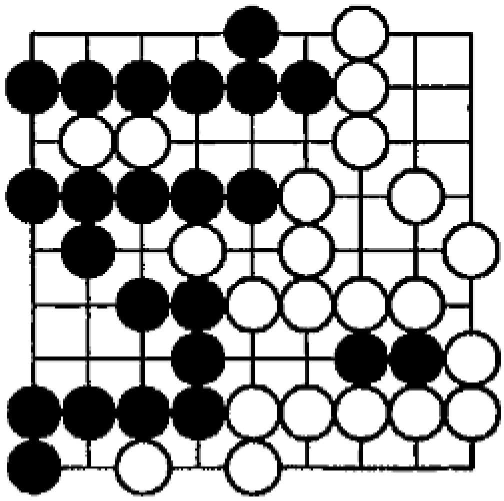


Figure C.21: *Hot miai*

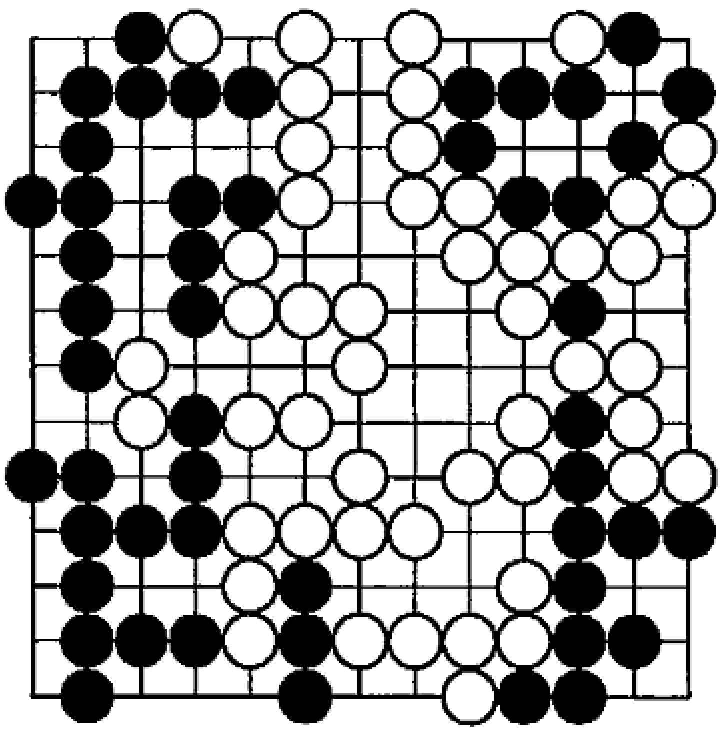


Figure C.22: *Two switches*