Robert Jasiek, research winter 2017/2018, edited extract.

Non-Existence of Local Double Sente

Game Tree

```
......A......
...../.\.....
..../...\....
...B.....C...
../.\.../.\..
.l...b.w...r.
```

Remarks

C_{GOTE} is a gote count. C_{SENTE} is a sente count. $\overline{C}$ is a white-count.

M_{GOTE} is a gote move value. $M_{B,SENTE}$ is Black's sente move value. $M_{W,SENTE}$ is White's sente move value. $\underline{M}$ is a tentative move value. F_B is Black's follow-up move value. F_W is White's follow-up move value.

P_A is the position of the following game tree's node A etc.

[Proposition 11 and related propositions say that Black's local sente is characterised by these equivalent, alternative value conditions:

$C_{SENTE} < C_{GOTE} \Leftrightarrow$

$M_{GOTE} < F_B \Leftrightarrow M_{B,SENTE} < F_B \Leftrightarrow M_{B,SENTE} < M_{GOTE}$.

Analogue move value conditions exist for White's local sente. See also the book *Endgame 3 - Values.*]

According to proposition 11, a local endgame is Black's local sente iff $C_{SENTE} < C_{GOTE}$. As a corollary, a local endgame is White's local sente iff $\overline{C}_{SENTE} < \overline{C}_{GOTE}$. We have $\overline{C}_{SENTE} < \overline{C}_{GOTE} \Leftrightarrow$ Csente > Cgote.

By proposition 11, $b < C_{GOTE}$ identifies Black's local sente and $-w < \overline{C}_{GOTE}$ identifies White's local sente so both conditions together identify a local double sente in proposition 16. The elegance of the theorem lies in the use of C_{GOTE} without spelling it out in detail in the form (X + Y) / 2. This simplifies the proof. Bill Spight has suggested to study the case b > w separately.

Proposition 16 does not make requirements for the lengths of the sequences to the followers but use the presupposition (6) so the proposition also expresses non-existence of long double sentes with long sequences. However, if some of l, b, w, r are unsettled positions instead of numbers, non-existence of local double sente is not proven yet by propositions 16 to 18. Future research should study this and whether they can be modified to allow basic endgame kos before encores.

Bill Spight: "As long as the results of alternating sequences are numbers, and they stop when a number is reached, the count of a (finite) combinatorial (non-ko) game lies between the result of the sequence when Black plays first and the result of the sequence when White plays first. If the two are equal to a number, n, then the game is equal to n. If the result when Black plays first is less than the result when White plays first, then the game is also a number which lies between the two results. If the result when Black plays first is greater than the result when White plays first, then the game cannot be a double sente, or it would have an infinite temperature. Anyway, the length of the sequences does not matter, as long as you stop when the result is a number." Bill Spight's reasoning should be formulated as a theorem and proof with references to applied theorems of combinatorial game theory. Then, non-existence of local double sente is proven for these assumptions: no kos, not doubly ambiguous, alternating sequences to settled positions, siblings of their leaves are settled positions.

Presuppositions for Propositions 16 to 18

After deleting dominated options and reversing reversible sequences, let the local endgame P_A have the black follower P_B and white follower P_C, let P_B have the black child l and white child b, let P_C have the black child w and white child r, with numbers[0)] l, b, w, r and l > b and w > r. The count and move value of P_A depend on C_B, C_C, b or w.[6)]

Proposition 16 [non-existence of a local double sente with b > w]

A local endgame does not exist without kos, with the black sente follower's count b, the white sente follower's white-count -w, b > w, b < Cgote and -w < $\overline{C}_{GOTE}$.

Proof

Proof by contradiction: Assume such a local endgame exists.

$-w < \overline{C}_{GOTE} \iff w > C_{GOTE}$. Together with the presupposition $b < C_{GOTE}$, this implies $b < C_{GOTE} < w$. This contradicts the presupposition $b > w$ and therefore such a local endgame does not exist.

Remarks

By the definition of sente count, b is Black's sente count ($C_{SENTE} = b$) and -w is White's sente white-count ($\overline{C'}_{SENTE} = -w$). C'_{SENTE} is not C_{SENTE} but occurs during a second application of proposition 11 in its variant as the aforementioned corollary. Proposition 16 and its proof do not use sente counts, proposition 11 and its corollary for White explicitly but proposition 16 interpreted in their context expresses non-existence of a local double sente with b > w.

Proposition 17 [non-existence of a local double sente with $b \leq w$]

A local endgame that is a local double sente does not exist without kos, with the black sente follower's count b, the white sente follower's count w, $b \leq w$.

Proof

Case I: $0 \leq b < w$:

The starting Black achieves b. The starting White achieves w. Since White's start is more favourable for Black than Black's start, Black wants to pass and let White start. Since White's starting play would result in a positive count $0 < w$ favouring Black, White passes, lets Black start and achieve the smaller (because $b < w$) non-negative count b. Hence, the local endgame is Black's local sente.

Case II: $b = w$:

Let $l := b + x$, $r := w - y$ with $x, y > 0.^{1)}$ (If $l = b$ or $w = r$, then P_B or P_C is a number so P_A is no local double sente.) We have $C_B =^{(0)} (l + b) / 2 =^{(1)} (b + x + b) / 2 = b + x/2$, $C_C =^{(0)} (w + r) / 2 =^{(II)} (b + r) / 2 =^{(1)(II)} (b + b - y) / 2 = b - y/2.^{2)}$

$\underline{M}_{GOTE} = (C_B - C_C) / 2 =^{(2)} ((b + x/2) - (b - y/2)) / 2 = x/4 + y/4$, $\underline{M}_{B,SENTE} = b - C_C =^{(2)} b - b + y/2 = y/2$, $\underline{M}_{W,SENTE} = C_B - w =^{(II)} C_B - b =^{(2)} b + x/2 - b = x/2.^{3)}$

Assume existence of a local double sente, that is $\underline{M}_{GOTE} > \underline{M}_{B,SENTE}, \underline{M}_{W,SENTE}$. We get

$\underline{M}_{GOTE} > \underline{M}_{B,SENTE} \iff^{(3)} x/4 + y/4 > y/2 \iff x/4 > y/4 \iff x > y^{4)}$,

$\underline{M}_{GOTE} > \underline{M}_{W,SENTE} \iff^{(3)} x/4 + y/4 > x/2 \iff y/4 > x/4 \iff y > x^{5)}$.

As (4) contradicts (5), a local double sente does not exist.

Case III: $b < 0$:

Even with $\underline{M}_{GOTE} > \underline{M}_{B,SENTE}, \underline{M}_{W,SENTE}$, we also have $b - w < 0$ (by the proposition's assumption $b \leq w$) so White prefers to pass instead of using his loss-making sente sequence. Therefore, we do not have a local double sente.

Remarks

By the definitions of Black's simple sente (with the key condition $M_{GOTE} > M_{B,SENTE}$), White's simple sente ($M_{GOTE} > M_{W,SENTE}$) and doubly ambiguous ($M_{GOTE} = M_{B,SENTE} = M_{W,SENTE}$), the latter cannot be a 'double sente', either. Due to the proposition's assumption $b \leq w$, cases I to III are a complete case analysis. In case I of the proof of proposition 17, since $P_A = \{b|w\}$ and not $b \geq w$, P_A is a number. Bill Spight did a preliminary study of examples, from which the reasoning of cases I and III are generalised here with variables.

Proposition 18 [non-existence of a local double sente]

A local endgame does not exist without kos, with the black sente follower's count b, the white sente follower's white-count -w, $b < C_{gote}$ and $-w < \overline{C}_{GOTE}$.

Proof

This corollary follows from proposition 16 combined with proposition 17 applied to the specific case $b < C_{gote}$ and $-w < \overline{C}_{GOTE}$.